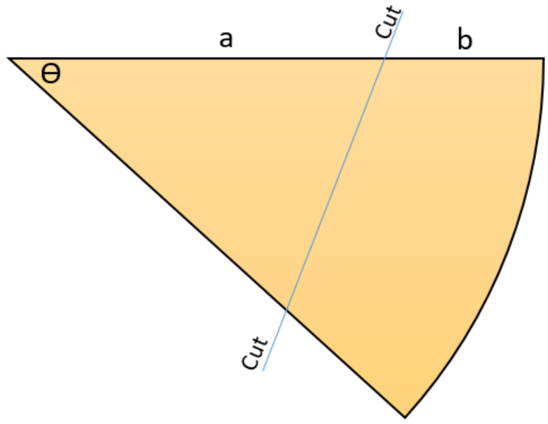


Let the knife cut the radii into two lengths in the ratio a:b. Let the angle subtended at the centre of the original cake be ϴ.

The cake is now in two pieces, both prisms, but let us consider just the area of their cross sections. One is an isosceles triangle and the other can be considered to be a sector of a circle minus that same isosceles triangle. If the two pieces have an equal area then:

Area of triangle = Area of sector – Area of triangle

2 x Area of triangle = Area of sector

$$2 \times \frac{1}{2} \times a \times a \times \sin\theta = \frac{\theta}{360} \times \pi \times (a+b)^2$$

$$a^2 \sin\theta = \frac{\theta\pi(a+b)^2}{360}$$

As we need to find the ratio of a:b we can consider the case where a=1

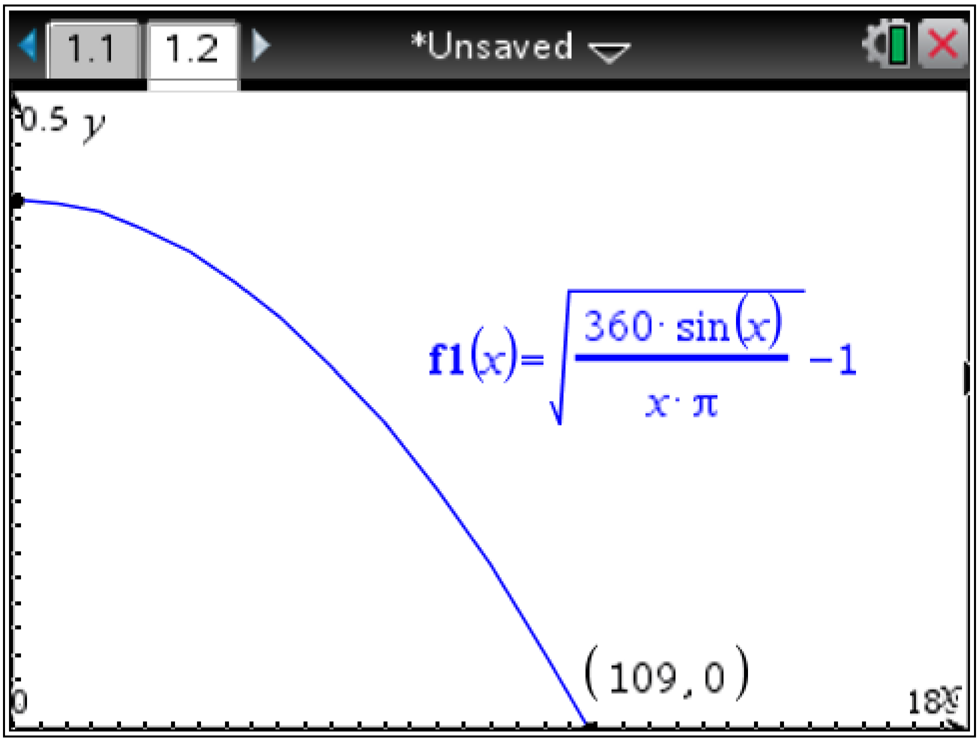
$$360\sin\theta = \theta\pi(1+b)^2$$

$$b = \sqrt{\frac{360\sin\theta}{\theta\pi}} - 1$$

So the value of b is dependent on the value of the angle of the sector.

This would seem to make sense as the case where ϴ=0 and ϴ=180 there would be no solutions.

A graph can be plotted showing how b (shown as y) varies as ϴ (shown as x) varies.



Values of b can be found for 0 < b < 109

The maximum value of b is $\sqrt{2}-1$ which would give a a:b ratio of

$$1:\sqrt{2}-1$$