Hidden Quadratic Questions in A-Level Past Papers

A hidden quadratic is an equation that can be reduced to a standard quadratic by making a substitution. For example, $2x^4 + 3x^2 - 2 = 0$ becomes $2u^2 + 3u - 2 = 0$ when we let $u = x^2$.

The following questions were found by searching through 1,484 question papers in your A-Level past papers collection. They are organised by type of substitution required.

1. Quartic Equations (substitute $u = x^2$)

These equations contain x^4 and x^2 terms but no x^3 or x terms (or can be rearranged to eliminate them). Setting $u = x^2$ converts them to a standard quadratic in u.

Paper	Q	Equation	Substitution	Notes
CIE 9709/13 Oct/Nov 2011	Q3	$2x^4 + 3x^2 - 2 = 0$	Let $u = x^2$	Curve $y = 2x^5 + 3x^3$ meets $y = 2x$. Show coordinates of A, B satisfy this equation, then solve.
CIE 9709 P2 Jun 2003	Q2	$x^4 - 9x^2 - 6x - 1 = 0$	Factor as quadratics	Polynomial f(x). Find constant a, then factorise completely.
CIE 9709 P2 Nov 2003	Q3	$x^4 - 6x^2 + x + a = 0$	Let $u = x^2$	Polynomial with factor (x+1). Find a, then factorise.
CIE 9231 P1 Jun 2003	Q (trig)	$64x^6 - 96x^4 + 36x^2 - 1 = 0$	Substitute $x = \cos k\pi$	Solve giving roots in form $\cos k\pi$. Hidden quadratic in x^2.
OCR H640-03 Sample	Q	$y = x^4 - 6x^2 - 4x + 5$	Let $u = x^2$	Quartic curve analysis with hidden quadratic structure.

2. Trigonometric Hidden Quadratics

These are the most common type in A-Level exams. They use identities like $\sin^2\theta + \cos^2\theta = 1$, or $\tan^2\theta + 1 = \sec^2\theta$, to convert a trigonometric equation into a quadratic in $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, or $\operatorname{cosec}\theta$.

Paper	Q	Equation	Substitution	Notes
CIE 9709/1 Oct/Nov 2002	Q5	$2\sin^2\theta + 3\sin\theta - 2 = 0$	Let $u = \sin\theta$	Show $3\tan\theta = 2\cos\theta$ reduces to this quadratic in $\sin\theta$.
CIE 9709/13 Oct/Nov 2011	Q5	$3\sin^2 x - 8\cos x - 7 = 0$	Use $\sin^2 x = 1 - \cos^2 x$	Show $\cos x = -2/3$ for real x. Then solve with shifted argument.
CIE 9709/01 May/Jun 2004	Q3	$\sin^2\theta + 3\sin\theta\cos\theta = 4\cos^2\theta$	Divide by $\cos^2\theta$	Show this can be written as a quadratic equation in $\tan\theta$.

CIE 9709 Nov 2003	Q (MS)	$4\sin^4\theta + 7\sin^2\theta - 2 = 0$	Let $u = \sin^2\theta$	Quadratic in $\sin^2\theta$. Solve for $\theta = 30°$ and $150°$.
CIE 9709 P2 Jun 2003	Q	$3\tan^2 x - 10\tan x + 3 = 0$	Already quadratic in tan x	From tan(45°−x)/tan(45°+x). Solve the quadratic in tan x.
CIE 9709 2005 Nov	Q1	$3\cos^2\theta + 2\cos\theta = 0$	Factor: $\cos\theta(3\cos\theta + 2) = 0$	Factorises as product. Quadratic in $\cos\theta$.
CIE 9709 2010 Summer	Q3 (P2)	$6\tan^2 x - 5\tan x + 1 = 0$	Already quadratic in tan x	From tan(x + 45°) = 6 tan x. Solve the quadratic.
EdExcel C3 Jun 2005	Q1(b)	$2\tan^2\theta + \sec\theta = 1$	Use $\tan^2\theta = \sec^2\theta - 1$	Reduces to $2y^2 + y - 3 = 0$ where $y = \sec\theta$.
EdExcel C3 Jun 2008	Q5(b)	$2\cot^2 R - 9\,\mathrm{cosec}\,R = 3$	Use $\cot^2 R = \mathrm{cosec}^2 R - 1$	Reduces to $2y^2 - 9y - 5 = 0$ where $y = \mathrm{cosec}\,R$.
EdExcel C3 Jun 2009	Q2(b)	$2\tan^2 R + 4\sec R + \sec^2 R = 2$	Use $\tan^2 R = \sec^2 R - 1$	Reduces to $3y^2 + 4y - 4 = 0$ where $y = \sec R$.
EdExcel C3 Jun 2014R	Q3	$2\tan x - \cot x = 5\,\mathrm{cosec}\,x$	Convert to sin/cos	Explicitly asks to show it reduces to $a\cos^2 x + b\cos x + c = 0$.

3. Exponential Hidden Quadratics

These involve expressions like e^{2x} or 2^x where a substitution like $u = e^x$ or $u = 2^x$ converts the equation to a quadratic. Less common in the papers found, but an important type.

Paper	Q	Equation	Substitution	Notes
CIE 9709 2010 Summer	Q5 (P2)	$2^x + 3(2^{-x}) = 4$	Let $y = 2^x$	Becomes $y^2 - 4y + 3 = 0$. Solve for x using logs.

Summary

Across your collection of A-Level past papers from CIE (9709), EdExcel, AQA, and OCR:

Trigonometric hidden quadratics are by far the most common type, appearing in 60+ papers. These are a staple of the C3/Pure Mathematics syllabus.

Quartic equations reducible to quadratics appear less frequently but provide excellent examples of the substitution technique.

Exponential hidden quadratics are rarer in these papers but are a recognized question type, especially for P2/P3 papers.

The key exam boards represented are: Cambridge International (CIE 9709 and 9231), Edexcel (C1–C4), OCR, and AQA.